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New non-Gaussian feature in COBE-DMR Four Year Maps

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ABSTRACT

We extend a previous bispectrum analysis of the Cosmic Microwave Background temperature anisotropy, allowing for the presence of correlations between different angular scales. We find a strong non-Gaussian signal in the "inter-scale" components of the bispectrum: their observed values concentrate close to zero instead of displaying the scatter expected from Gaussian maps. This signal is present over the range of multipoles $\ell = 6 - 18$, in contrast with previous detections. We attempt to attribute this effect to galactic foreground contamination, pixelization effects, possible anomalies in the noise, documented systematic errors studied by the COBE team, and the effect of assumptions used in our Monte Carlo simulations. Within this class of systematic errors the confidence level for rejecting Gaussianity varies between 97% and 99.8%.

Subject headings: Cosmology: cosmic microwave background – theory – observations

1. Introduction

In a recent *Letter* Ferreira, Magueijo & Górski (1998) found strong evidence for non-Gaussianity in the anisotropy of the Cosmic Microwave Background (CMB) temperature. This detection was followed by similar claims from Novikov, Feldman and Shandarin (1998) and Pando, Valls-Gabaud & Fang (1998), and caused considerable consternation among theorists (see Kamionkowski and Jaffe 1998 for a discussion). These three groups employed very different statistical tools, but used the same dataset – the COBE-DMR 4 year maps. Recent work by Bromley & Tegmark 1999 confirmed these measurements, but Banday, Zaroubi & Górski 99 cast doubts upon the cosmological origin of the observed signals. We clearly do not fully understand some of the less conspicuous systematic errors associated with DMR maps. We feel that the origin of the observed non-Gaussian features will probably not be conclusively identified before an independent all-sky dataset becomes available.

In this Letter we revisit and complete the analysis of Ferreira, Magueijo & Górski (1998). In that work the possibility of departures from Gaussianity was examined in terms of the bispectrum. Given a full-sky map, $\frac{\Delta T}{T}(\mathbf{n})$, this may be expanded into Spherical Harmonic functions:

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\mathbf{n}) \tag{1}$$

The coefficients $a_{\ell m}$ may then be combined into rotationally invariant multilinear forms (see Magueijo, Ferreira, and Górski 1998 for a possible algorithm). The most general cubic invariant is the bispectrum, and is given by

$$\begin{aligned} \hat{B}_{\ell_1\ell_2\ell_3} &= \alpha_{\ell_1\ell_2\ell_3} \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \\ \alpha_{\ell_1\ell_2\ell_3} &= \frac{1}{(2\ell_1+1)^{\frac{1}{2}}(2\ell_2+1)^{\frac{1}{2}}(2\ell_3+1)^{\frac{1}{2}}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1} \end{aligned} \tag{2}$$

where the $(\ldots)$ is the Wigner $3J$ symbol. In Ferreira, Magueijo & Górski (1998) correlations between different multipoles were ignored, and so $\ell_1 = \ell_2 = \ell_3$. Here we consider bispectrum components sensitive to correlations between different scales. Selection rules require that

$\ell_1+\ell_2+\ell_3$ be even. The simplest chain of correlators is therefore $\hat{A}_\ell = B_{\ell-2\,\ell\,\ell+2}$, with ℓ even. Other components, involving more distant multipoles, may be considered but they are very likely to be dominated by noise; it is natural to assume that possible non-Gaussian inter-scale correlations decay with ℓ separation. We shall consider a ratio

$$J^3_\ell = \frac{\hat{A}_\ell}{(\hat{C}_{\ell-2})^{1/2}(\hat{C}_\ell)^{1/2}(\hat{C}_{\ell+2})^{1/2}} \tag{3}$$

where $\hat{C}_\ell = \frac{1}{2\ell+1}\sum_m |a_{\ell m}|^2$. This quantity is dimensionless, and is invariant under rotations and parity. It extends the I^3_ℓ statistic used in Ferreira, Magueijo & Górski (1998) [1].

The theoretical importance of the bispectrum as a non-Gaussian qualifier has been recognized in a number of publications (Luo 1994, Peebles 1998, Spergel & Goldberg 1998, Goldberg & Spergel 1998, Wang & Kamionkowski 99). Kogut et al. 1996 measured the pseudocollapsed and equilateral three point function of the DMR four year data. The bispectrum may be regarded as the Fourier space counterpart of the three point function. The work presented in this *Letter*, combined with Ferreira, Magueijo & Górski (1998) and Heavens 1998, provides a complete set of signal dominated bispectrum components inferred from DMR maps.

2. The publicly released 4 year maps

We first consider the inverse-noise-variance-weighted, average maps of the 53A, 53B, 90A and 90B *COBE*-DMR channels, with monopole and dipole removed, at resolution 6, in galactic and ecliptic pixelization. We use the extended galactic cut of Banday *et al* 1997, and Bennet *et al* 1996 to remove most of the emission from the plane of the Galaxy. To estimate the J^3_ℓs we set the value of the pixels within the galactic cut to 0 and the monopole and dipole of the cut map to zero. We then integrate the map multiplied with spherical harmonics to obtain the estimates of the $a_{\ell m}$s

[1]Taking the modulus is not necessary to ensure parity invariance, contrary to the statement made in Ferreira, Magueijo & Górski (1998). This does not affect any of the conclusions, since $P(I^3_\ell) = P(-I^3_\ell)$ for a Gaussian process.

and apply equations 2 and 3. The observed J^3_ℓ are to be compared with their distribution $P(J^3_\ell)$ as inferred from Monte Carlo simulations in which Gaussian maps are subject to DMR noise and galactic cut. In simulating DMR noise we take into account the full noise covariance matrix, as described in Lineweaver et al 1994. This includes correlations between pixels 60° degrees apart.

The results are displayed in Fig. 1. Leaving aside the deviant J^3_4, it is blatantly obvious that the observed J^3_ℓ do not exhibit the scatter around zero implied by $P(J^3_\ell)$ in the range of scales $\ell = 6 - 18$. This may be mathematically formalized by means of a goodness of fit statistic, such as the "chi squared":

$$X^2 = \frac{1}{N}\sum_\ell X^2_\ell = \frac{1}{N}\sum_\ell(-2\log P_\ell(J^3_\ell) + \beta_\ell), \tag{4}$$

where the constants β_ℓ are defined so that for each term of the sum $\langle X^2_\ell\rangle = 1$. As explained in Ferreira, Magueijo & Górski (1998) this quantity reduces to the usual chi squared when the distributions P are Gaussian. When P is not Gaussian, X^2 goes to infinite where P goes to zero, reaches a minimum (usually around zero) at the peak of P, and has average 1. Hence X^2 does for a non-Gaussian P what the usual chi squared does for a Gaussian P. A good fit is represented by $X^2 \approx 1$. If $X^2 \gg 1$ the data is plagued by deviants, that is observations far in the tail of the theoretical distribution. If $X^2 \ll 1$ the observations fail to exhibit the scatter predicted by the theory, concentrating uncannily on the peak of the distribution. Both cases present grounds for rejecting the hypothesis embodied in $P(J^3_\ell)$, in our case Gaussianity.

We find $X^2 = 0.14$ and $X^2 = 0.22$ for data in galactic and ecliptic pixelization, respectively. To quantify the confidence level for rejecting Gaussianity we determine the distribution of X^2, $F(X^2)$, making use of further Monte Carlo simulations. The detailed procedure shadows that described in Ferreira, Magueijo & Górski (1998). We stress that in each realization a new sky is produced, from which a full set of J_ℓ is derived, for which the X^2 is computed. The result is plotted in Fig. 2, where we superimpose the observed X^2 and its distribution. We then compute the percentage of the population with a larger X^2 than the observed one. We find that $P(X^2 > 0.14) = 0.998$ (and $P(X^2 > 0.22) = 0.985$) for maps in galactic (ecliptic) pixelization. The lack of scatter in the observed J^3_ℓ implies that Gaussianity may be rejected at the 99.8%

(98.5%) confidence level.

A closer analysis reveals that this signal is mainly in the 53 GHz channel (see Table 1), which is also the least noisy channel. However the confidence level for rejecting Gaussianity increases (from 93.2% to 99.8% in galactic pixelization) when the 53Ghz channel is combined with the 90Ghz channel. Hence the overall signal is due to both channels. The reduced confidence levels in the separate channels merely reflect a lower signal to noise ratio, and the Gaussian nature of noise.

3. Combining different Gaussianity tests

Woven into the above argument is a perspective on how to combine different Gaussianity tests which differs from that presented by Bromley & Tegmark 1999. In that work the authors argue that if n tests are made for a given hypothesis, and they return confidence levels for rejection $\{p_i\}$, then, if $p_{max} = \max\{p_i\}$, the actual confidence level for rejection is p^n_{max}.

While the above recipe is formally correct it cannot be applied when the hypothesis is Gaussianity. Let the various tests be a set of cumulants $\{\kappa_i\}$ (Stuart & Ord 1994). Suppose that all cumulants are consistent with Gaussianity except for a single cumulant, which prompts us to reject Gaussianity with confidence level p_{max}. Clearly the confidence level for rejecting Gaussianity is p_{max}, since it is enough for the distribution to have a single non-Gaussian cumulant for it to be non-Gaussian.

The point is that Gaussianity cannot be regarded by itself as an hypothesis, since the corresponding alternative hypothesis includes an infinity of independent degrees of freedom involving different moments and scales. The argument of Bromley & Tegmark 1999 is correct when applied to independent tests concerning the same non-Gaussian degree of freedom, for instance independent tests related to the skewness. However it cannot be true for different tests probing independent non-Gaussian degrees of freedom, say skewness and kurtosis.

In the context of our result (which returns $X^2 \ll 1$), we notice that if we were to include into the analysis the results of Ferreira, Magueijo & Górski (1998) (for which $X^2 \gg 1$) we would get an average $X^2 \approx 1$. Such procedure is obviously nonsensical: two wrongs don't make a right. One should examine independent non-Gaussian features separately, in particular the I^3_ℓ and the J^3_ℓ, or two ranges of ℓ, one with $X^2 \gg 1$ the other with $X^2 \ll 1$. The only practical constraint is sample variance, forcing any analysis to include more than one degree of freedom so that $F(X^2)$ is sufficiently peaked.

4. The possibility of a non-cosmological origin

Could this signal have a non-cosmological origin? The possibility of foreground contamination was considered in two ways. Firstly we subject foreground templates to the same analysis. At the observing frequencies the obvious contaminant should be foreground dust emission. The DIRBE maps (Boggess et al. 1992) supply us with a useful template on which we can measure the J^3_ℓs. We have done this for two of the lowest frequency maps, the 100 μm and the 240 μm maps. The estimate is performed in exactly the same way as for the DMR data (i.e. using the extended Galaxy cut). We performed a similar exercise with the Haslam 408Mhz (Haslam (1982)) map. The results are presented in Table 1. We find that the J^3_ℓ (in contrast to the I^3_ℓ used by Ferreira, Magueijo & Górski (1998)) are capable of exposing the non-Gaussianity in these templates, even when smoothed by a 7° beam. However none of the signatures found correlates with the DMR signal. DIRBE maps produce highly deviant J^3_ℓ, whereas all $J^3_\ell > 0$ for the Haslam map.

We also considered foreground corrected maps (see Table 1). In these one corrects the coadded 53 and 90 Ghz maps for the DIRBE correlated emission. We studied maps made in ecliptic and galactic frames, and also another map made in the ecliptic frame but with the DIRBE correction forced to have the same coupling as determined in the galactic frame. The confidence levels for rejecting Gaussianity are 97.9%, 98.5%, and 97.1%, respectively. The signal is therefore reduced, but not erased.

Could the observed signal be due to detector noise? The DMR noise is subtly non-Gaussian, due to its anisotropy and pixel-pixel correlations (Lineweaver et al 1994). These features were incorporated into the simulations leading to $P(J^3_\ell)$. However it could just happen that the noise in the particular realization we have observed turned out to be a fluke, concentrating the observed J^3_ℓ around zero. We examined this possibility by considering difference maps $(A-B)/2$. If the observed effect is the result of a noise fluke, it should be exacerbated in $(A-B)/2$ maps, rather than in $(A+B)/2$ maps. As can be seen in Table 1, one of the noise maps ($(A-B)/2$, 53Ghz, in ecliptic pixelization) is indeed unusually non-Gaussian — but with $X^2 \gg 1$ rather than $X^2 \ll 1$. This feature disappears in $(A-B)/2$, 53Ghz, maps made in the galactic pixelization. The $(A-B)/2$, 90GHz map, in galactic pixelization, has a low X^2 but far from significant.

Bromley & Tegmark 1999 have shown that removing a selected beam-size region from DMR maps deteriorates the I^3_ℓ non-Gaussian signal. This fact is of great interest as it highlights the possible spatial localization of what is a priori a "Fourier space" statistic. More recently Banday, Zaroubi & Górski 99 pointed out that removing single beam-size regions may also reinforce the I^3_ℓ non-Gaussian signal. We have subjected our J^3_ℓ analysis to this exercise. We found that X^2 from maps without a single beam sized region is *very* sharply peaked around the uncut value 0.14. There is a region without which $X^2 = 0.22$ but it is also possible to remove a region so that $X^2 = 0.08$. Hence the J^3_ℓ signal can never be significantly deteriorated by means of this prescription, and for this reason we believe it to be essentially a Fourier space feature.

A number of systematic error templates were also examined (Kogut et al. 1996b). These provide estimates at the 95% confidence level of errors due to the following: the effect of instrument susceptibility to the Earth magnetic field; any unknown effects at the spacecraft spin period; errors in the calibration associated with long-term drifts, and calibration errors at the orbit and spin frequency; errors due to incorrect removal of the COBE Doppler and Earth Doppler signals; errors in correcting for emissions from the Earth, and eclipse effects; artifacts due to uncertainty in the correction for the correlation created by the low-pass filter on the lock-in amplifiers (LIA) on each radiometer; errors due to emissions from the moon, and the planets. The systematic templates display strongly non-Gaussian structures, tracing the DMR scanning patterns. We added or subtracted these templates enhanced by a factor of up to 4 to DMR maps (see Magueijo, Ferreira, and Górski 1998 for a better description of the procedure). The effect on the J_ℓ^3 spectrum was always found to be small, leading to very small variations in the X^2.

Banday, Zaroubi & Górski 99 have recently claimed that the systematic errors due to eclipse effects may be larger than previously thought. They showed how the I_ℓ^3 change dramatically when estimated from maps in which data collected in the two month eclipse season has been discarded. These maps are more noisy, and the I_ℓ^3 are very sensitive to noise. Indeed the I_ℓ^3 are cubic statistics, with a signal to noise proportional to (Number of Observations)$^{3/2}$, and so they are much more sensitive to noise than the power spectrum. Perhaps the variations in I_ℓ^3 merely reflect a larger noise, and not a systematic effect. This possibility could be disproved if no striking variations in the I_ℓ^3 were found in maps for which *other* two month data samples are excised. We have applied the J_ℓ^3 analysis to maps without eclipse data, and found that the confidence level for rejecting Gaussianity does not drop below 99.2% (see Table 1). Hence the result described in this Letter appears to be robust in this respect. A more detailed description of the impact of systematics upon the I_ℓ^3 and J_ℓ^3 will be the subject of a comprehensive publication (Banday, Ferreira, Górski, Magueijo, & Zaroubi 99).

We finally subject our algorithm to a number of tests. Arbitrary rotations of the coordinate system (as opposed to the pixelization scheme) affects J_ℓ^3 to less than a part in 10^5. Possible residual offsets (resulting from the removal of the monopole and dipole on the cut map) do not destroy the signal found. Finally changing the various assumptions going into Monte Carlo simulations do not affect the estimated distributions $P(J_\ell^3)$. We found these distributions to be independent of the assumed shape of the power spectrum, of the exact shape of the DMR beam, or the inclusion of the pixel window function.

In summary the J_ℓ^3 analysis appears to be more sensitive to shortcomings in DMR maps than the I_ℓ^3. This fact is already obvious in the differences between ecliptic and galactic pixelizations in the publicly released maps. When all possible renditions of DMR data are considered the significance level of our detection may vary between 97% and 99.8%. Therefore the various tests for systematics we have described do not leave the result unscathed, but neither do they rule out a cosmological origin. We should stress that, in line with all previous work in the field, we have employed a frequentist approach. One may therefore question the Bayesian meaning of the confidence levels quoted. A Bayesian treatment of the bispectrum remains unfeasible (see however Contaldi *et al* 99).

In comparison with Ferreira, Magueijo & Górski (1998) the result we have described is more believable from a theoretical point of view. It spreads over a range of scales. Previous detections concentrate on a single mode. The result obtained is puzzling in that, rather than revealing the presence of deviants, it shows a perfect alignment of the observed J^3_ℓ on the top of their distribution for a Gaussian process. This is perhaps not as strange as it might seem at first: in Ferreira, Magueijo & Silk 1997 it was shown how non-Gaussianity may reveal itself not by non-zero average cumulants, but by abnormal errorbars around zero.

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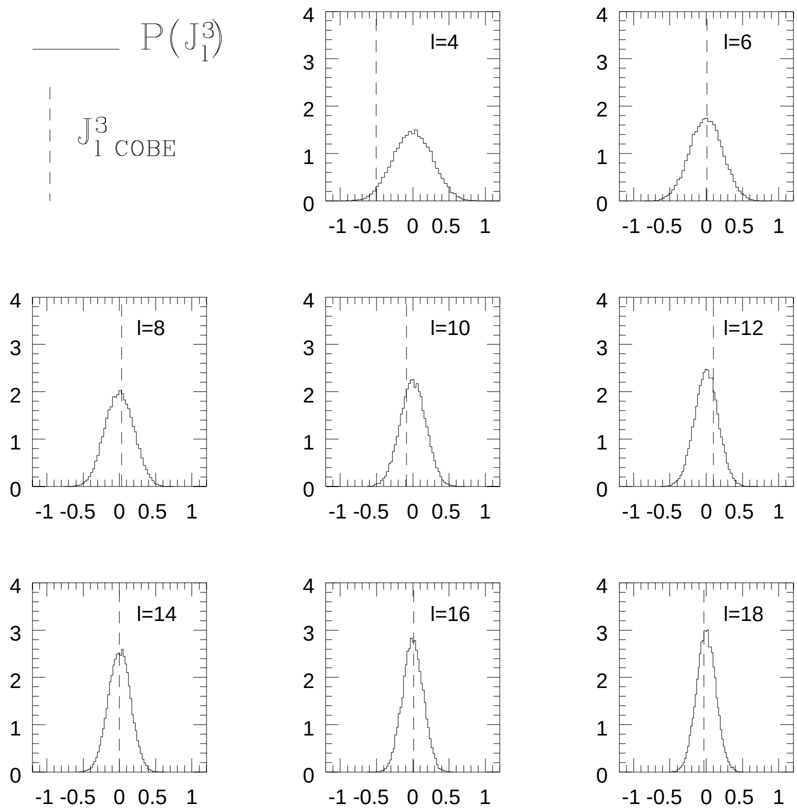

Fig. 1.— The vertical thick dashed line represents the value of the observed J_ℓ^3. The solid line is the probability distribution function of J_ℓ^3 for a Gaussian sky with extended galactic cut and DMR noise, as inferred from 25000 realizations.

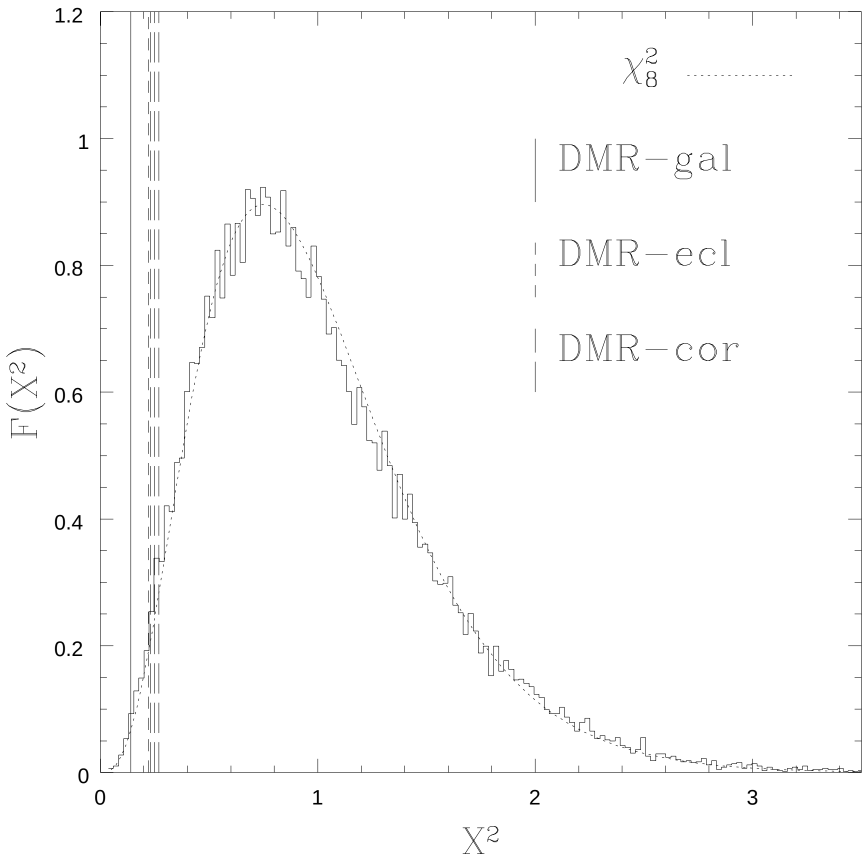

Fig. 2.— The distribution $F(X^2)$ as inferred from 25000 realizations. $F(X^2)$ is well approximated by a χ^2_8. The vertical bars show the observed X^2 in COBE-DMR 4 year maps in galactic and ecliptic pixelizations. We also show the result for the various foreground corrected maps - for which there is higher concordance.

	J^3_4	J^3_6	J^3_8	J^3_{10}	J^3_{12}	J^3_{14}	J^3_{16}	J^3_{18}	X^2	Reject %
Gauss-rms	.263	.225	.194	.178	.162	.153	.144	.135	—	—
DMR - ecl	-.521	.009	-.022	-.007	.112	.088	.054	.045	.22	98.5
DMR - gal	-.502	.012	.029	-.088	.098	-.002	.013	-.030	.14	99.8
DMR - cor/ecl	-.554	.022	-.091	-.099	.104	.061	.071	.057	.25	97.9
DMR - cor/ecg	-.555	.026	-.105	-.104	.102	.055	.076	.059	.27	97.1
DMR - cor/gal	-.542	.042	-.068	-.140	.108	-.026	.031	-.019	.23	98.5
DMR - gal 90	-.513	.003	-.316	-.167	.140	.063	.042	-.049	.66	71.2
DMR - gal 53	-.497	-.053	-.009	.022	.172	-.076	.107	-.031	.36	93.2
DMR - gal/ne	-.448	.034	.042	-.022	.127	.009	.008	.042	.18	99.2
A-B 53 ecl	-.304	-.002	.126	-.008	-.426	.437	.170	-.047	2.41	98.4 *
A-B 53 gal	-.259	.034	.118	-.003	-.193	.248	.246	.019	1.07	38.1
A-B 90 ecl	-.134	-.151	-.195	.099	-.181	.091	-.273	.021	1.00	43.3
A-B 90 gal	-.085	-.158	-.173	.001	-.140	.047	-.180	.079	.57	79.0
DIRBE08 ecl	.076	.203	-.109	.404	.240	.027	-.293	-.107	1.93	94.1*
DIRBE10 ecl	.252	.124	-.267	.706	.134	.184	-.177	-.145	3.15	99.7*
Haslam ecl	.279	.459	.062	.158	.196	.146	.049	.100	1.21	29.4
Haslam gal	.258	.462	.044	.182	.163	.105	.037	.045	1.04	39.7

Table 1: The values of J^3_ℓ for various datasets, their X^2, and the confidence level for rejecting Gaussianity on the grounds of $X^2 \ll 1$ (starred (*) figures indicate confidence levels for rejecting Gaussianity on the grounds of $X^2 \gg 1$). "ecl" and "gal" stand for ecliptic and galactic pixelizations, "cor" for foreground corrected, "ne" for no-eclipse data. On the first line we show the rms for a Gaussian process. This table is more illuminating than a plot, as most J^3_ℓ accumulate around zero.